When you finish your homework you should be able to...
- π Use Sigma Notation to Write and evaluate a Sum
- π Understand the Concept of Area
- π Approximate the Area of a Plane Region
- π Find the Area of a Plane Region Using Limits

Warm-up: Evaluate the following limits.

1. $\lim\limits_{x \to \infty} \dfrac{25x^2 - 5x + 3}{x^2}$

2. $\lim\limits_{x \to \infty} \left(\dfrac{2x^3 + 6x^2}{3x^3} + 10 \right)$

SIGMA NOTATION

The sum of n terms $a_1, \ldots, a_n$ is written as $\sum\limits_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$ where i is the **index of summation**, a_i is the **ith term** of the sum, and the **upper and lower bounds of summation** are n and 1.

Example 1: Find the sum.

a. $\sum\limits_{i=1}^{5} i$

b. $\sum\limits_{i=1}^{4} i^2$

c. $\sum\limits_{i=1}^{3} \sqrt{i}$

Summation Properties

1. $\displaystyle\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$

2. $\displaystyle\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

Example 2: Find the sum.

a. $\displaystyle\sum_{i=1}^{4} 2$

b. $\displaystyle\sum_{i=1}^{3}\left(1-i^2\right)$

Theorem: Summation Formulas

1. $\displaystyle\sum_{i=1}^{n} c = cn$

2. $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3. $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

4. $\displaystyle\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Example 3: Evaluate the following sums.

a. $\displaystyle\sum_{i=1}^{n}\left(3i-i^2\right)$

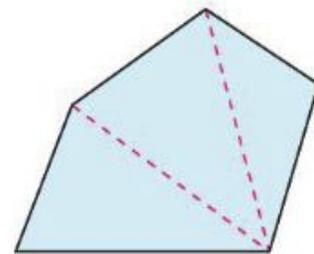b. $\displaystyle\sum_{i=1}^{n}\left(6i+4i^3\right)$

Area

In Euclidean geometry, the simplest type of plane region is a _____.
The definition for the area of a rectangle is _____.From this definition, you
can develop formulas for the areas of many other plane regions such as triangles.
To determine the area of a triangle, you can from a _____ whose
area is _____ that of the _____. Once you know how to
find the area of a triangle, you can determine the area of any _____
by subdividing the polygon into _____ regions.
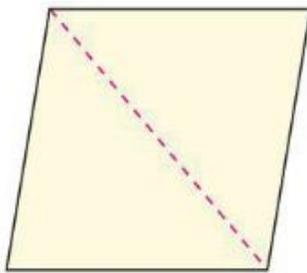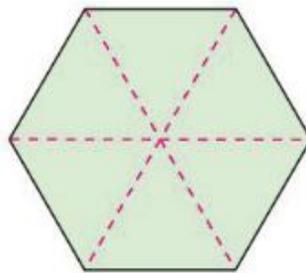
Parallelogram   Hexagon   Polygon
Figure 5.6

Finding the areas of regions other than polygons is more difficult. The ancient
Greeks were able to determine formulas for the areas of some general regions by
the _____ method. Essentially, the method is a _____
process in which the area is _____ between two polygons—one
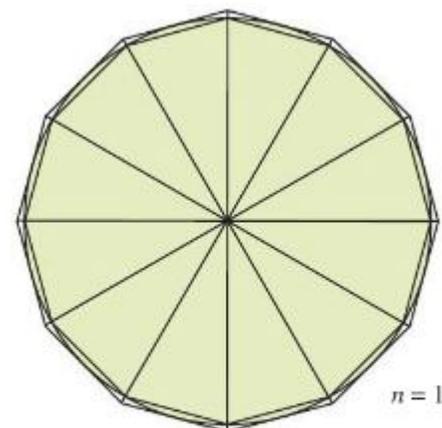_____ in the region and one _____ about
the region.
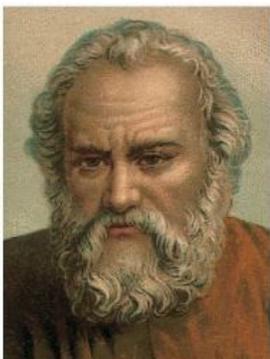
ARCHIMEDES (287–212 B.C.)

Archimedes used the method of
exhaustion to derive formulas for
the areas of ellipses, parabolic
segments, and sectors of a spiral.
He is considered to have been
the greatest applied mathematician
of antiquity.
See LarsonCalculus.com to read
more of this biography.
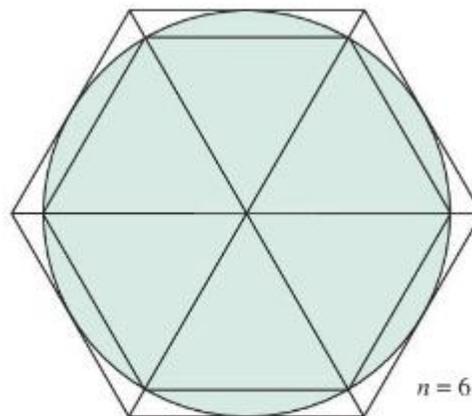
$n = 6$

$n = 12$

The exhaustion method for finding the area of a circular region
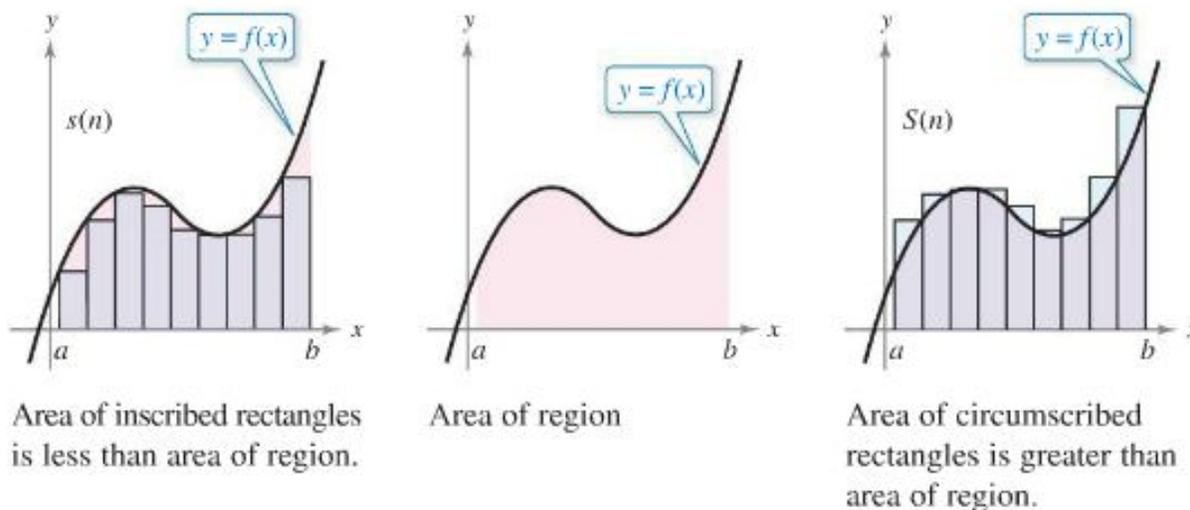**Figure 5.7**

$s(n) \le$ (Area of region) $\le S(n)$

Figure 5.11

Area of inscribed rectangles is less than area of region.

Area of region

Area of circumscribed rectangles is greater than area of region.

Theorem: Limits of the Lower and Upper Sums

Let f be continuous and nonnegative on the interval $[a,b]$. The limits as $n \to \infty$ of both the lower and upper sums exist and are equal to each other. That is,
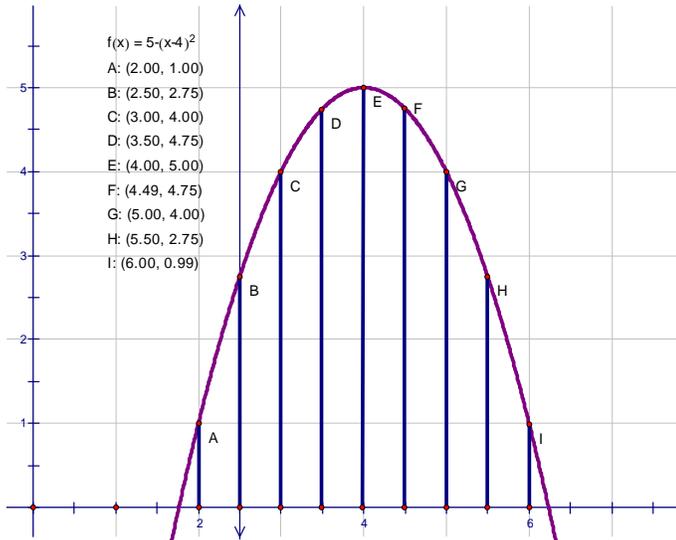
$$\lim_{n \to \infty} s(n) = \lim_{n \to \infty} \sum_{i=1}^{n} f(m_i)\Delta x$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} f(M_i)\Delta x$$

$$= \lim_{n \to \infty} S(n)$$

where $\Delta x = \dfrac{b-a}{n}$ and $f(m_i)$ and $f(M_i)$ are the minimum and maximum values of f on the subinterval.

Example 4:

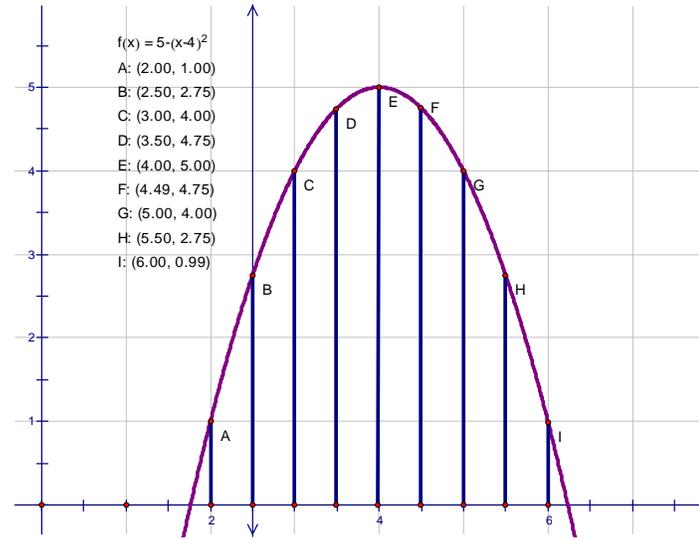a. Find the upper sum from

$x = 2$ to $x = 6$.

b. Find the lower sum from

$x = 2$ to $x = 6$.

$f(x) = 5-(x-4)^2$
A: (2.00, 1.00)
B: (2.50, 2.75)
C: (3.00, 4.00)
D: (3.50, 4.75)
E: (4.00, 5.00)
F: (4.49, 4.75)
G: (5.00, 4.00)
H: (5.50, 2.75)
I: (6.00, 0.99)

$f(x) = 5-(x-4)^2$
A: (2.00, 1.00)
B: (2.50, 2.75)
C: (3.00, 4.00)
D: (3.50, 4.75)
E: (4.00, 5.00)
F: (4.49, 4.75)
G: (5.00, 4.00)
H: (5.50, 2.75)
I: (6.00, 0.99)

Definition of an Area in the Plane

Let f be continuous and nonnegative on the interval $[a,b]$. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x=a$ and $x=b$ is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x, \qquad x_{i-1} \le c_i \le x_i \qquad \text{where } \Delta x = \frac{b-a}{n},$$

right endpoint: $c_i = a+i\Delta x$, left endpoint: $c_i = a+(i-1)\Delta x$

Example 5: Find the area of the region bounded by the graph $f(x) = x^3$, the x-axis, and the vertical lines $x=0$ and $x=1$.